MATHEMATICS SPECIALIST UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section One:
Calculator-free

WA student number:　　In figures

　　　　　　　　　　　In words _____

_____

　　　　　　　Your name _____

Time allowed for this section
Reading time before commencing work:　five minutes
Working time:　　　　　　　　　　　fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items:　pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:　nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
    Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	6	
2	5	
3	7	
4	9	
5	8	
6	7	
7	8	
S1 Total	50	
S1 Wt (×0.7)	35%	
S2 Wt	65%	
Total	100%	

See next page

Section One: Calculator-free 35% (50 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

The polynomial $f(z) = g(z) \times h(z)$, where $h(z) = z^2 - 4z + 5$.

(a) Show that $z - 2 - i$ is a factor of $h(z)$. (2 marks)

(b) Given that $f(z) = z^4 - 6z^3 + 17z^2 - 26z + 20$, solve $f(z) = 0$, giving all solutions in Cartesian form. (4 marks)

Question 2 **(5 marks)**

(a) Express the complex number $\dfrac{8}{1-\sqrt{3}i}$ in the form $r \operatorname{cis}\theta$, $-\pi < \theta \le \pi$. (3 marks)

(b) When $u = 4\operatorname{cis}\left(\dfrac{\pi}{12}\right)$ and $v = 5\operatorname{cis}\left(-\dfrac{\pi}{10}\right)$ determine

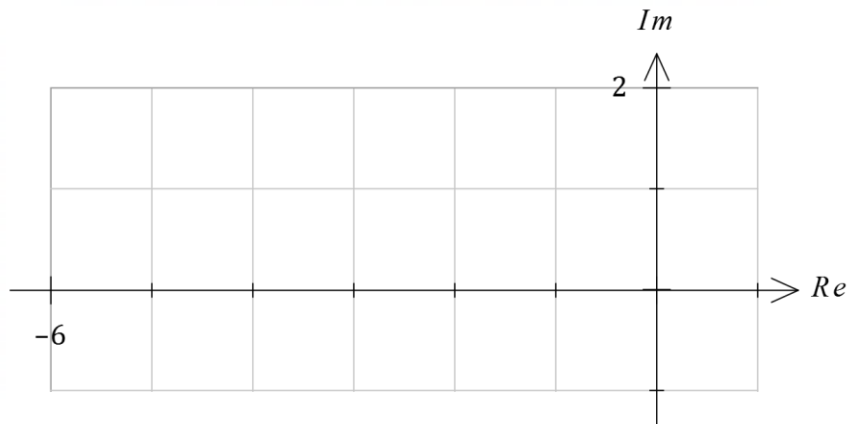(i) $|uv^3|$. (1 mark)

(ii) $\arg(v \div u)$. (1 mark)

Question 3 **(7 marks)**

Consider the complex number $z = -\sqrt{3} + i$.

(a) On the Argand diagram below, draw a line segment from the origin to z and from the origin to $z - 2$. (2 marks)

(b) Determine the principal value of the argument of $z - 2$. (3 marks)

(c) Determine the value of the modulus of $z - 2$. (2 marks)

Question 4 **(9 marks)**

(a)   Solve the following system of equations and interpret the solution geometrically.

(4 marks)

$$x - y + z = 7$$
$$x + 2y + 3z = -10$$
$$x - y - z = 9$$

(b)   The position vectors of points P, Q and R are $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OR} = \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$.

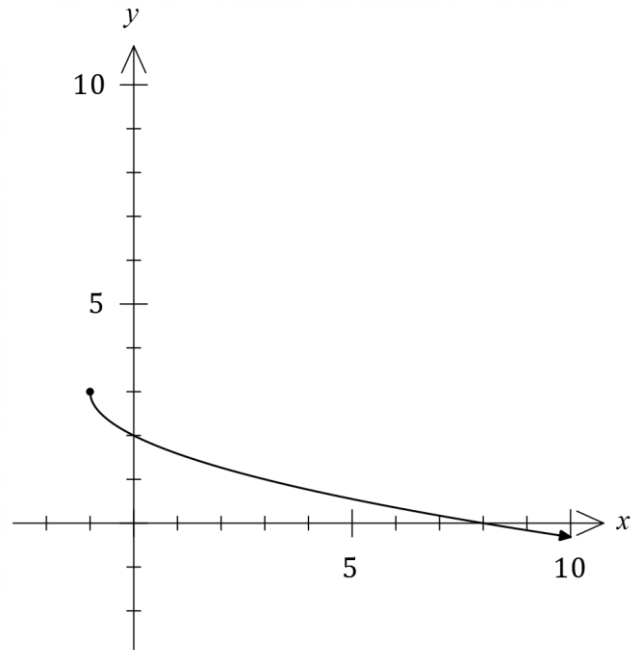Determine the Cartesian equation of the plane through line PQ and perpendicular to plane OQR. (5 marks)

See next page

Question 5 (8 marks)

Function f is defined as $f(x) = 3 - \sqrt{x+1}$.

The graph of $y = f(x)$ is shown at right.

(a) Sketch the graph of $y = f^{-1}(x)$ on the axes above. (2 marks)

(b) State the domain and range of $f^{-1}(x)$. (2 marks)

Function g is defined as $g(x) = \sqrt{x}$, and $h(x) = g \circ f(x)$.

(c) Write an expression for $h(x)$ and determine the domain and range of $h(x)$. (4 marks)

Question 6 **(7 marks)**

Let the complex number $v = \sqrt{2}\operatorname{cis}\left(-\dfrac{3\pi}{4}\right)$. Describe geometrically the locus of the complex number $z = x + iy$ in the Argand plane that is determined by the relation $\sqrt{2}|z - v^2| = |z - v|$.
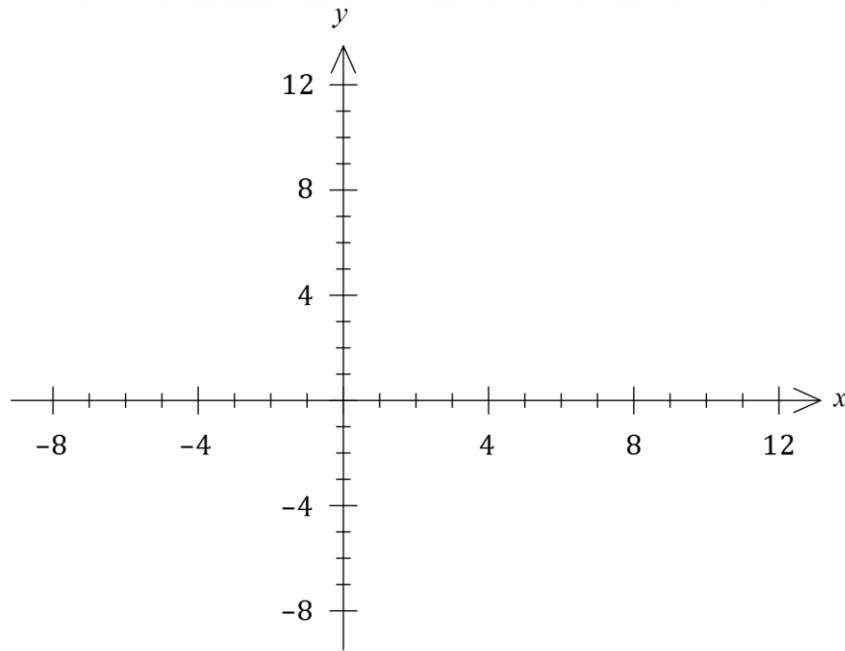
Question 7   **(8 marks)**

Consider the function $f(x) = \dfrac{x^2 + bx + c}{ax + d}$, where a, b, c and d are constants.

The graph of $y = f(x)$ has roots at $x = -2$ and $x = 3$, a vertical asymptote $x = 4$ and passes through the point $(5, 7)$.

Sketch the graph of $y = f(x)$, clearly showing the y-intercept and equations of all asymptotes.

End of questions

Supplementary page

Question number: _____